

Constructions of new simply connected surfaces of general type

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Chern numbers c_1^2, c_2 of a complex minimal surface of general type are topological invariants that satisfy the so called Bogomolov-Miyaoka-Yau inequality: $c_1^2 \leq 3c_2$. By a well-known result of Yau, a surface X satisfies the equality $c_1^2 = 3c_2$ if and only if its universal cover is the unit disc \mathbb{B}^2 . In that case the fundamental group of X is a discrete, torsion free and co-compact subgroup of $\mathrm{PU}(2,1)$, and such a surface is never simply connected. At the end of the 70's, Bogomolov asked if one can improve the Bogomolov-Miyaoka-Yau inequality by an inequality of the type $c_1^2 \leq Ac_2$ with $A < 3$ if one suppose moreover that X is simply connected. In this talk, we will explain how to construct simply connected surfaces of general type with c_1^2/c_2 arbitrarily close to 3. Thus the answer to Bogomolov's question is negative. Paradoxically, our construction uses surfaces which are quotient of the unit ball. This is a joint work with G. Urzua.